

EVALUATION AND SUBSTITUTION

Summary

When letters in a formula are replaced by numbers, it's called substitution.

EXAMPLES:

1. Find the value of $\sqrt{x^2 + y^2}$ when $x = 8$ and $y = -6$

2. Find the value of $x^2 - x + 6$ when $x = -3$

3. Find the value of $2(4 + 3y) + y(3 - 2t)$ when $y = 2$ and $t = 3$

4. Given that $P = 200$, $r = 50$ and $n = 2$, find the value of $A = P\left(1 + \frac{r}{100}\right)^n$

5. If $a = 6$, $b = 8$, $c = 10$ and $s = 12$, find the value of $A = \sqrt{s(s - a)(s - b)(s - c)}$

6. Given that $v^2 = u^2 + 2as$, find the values of v when $u = 6$, $a = 3.5$ and $s = 4$.

7. Given that $a = 2$, $b = -4$ and $c = -6$, find the values of $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

8. Given that $y = \frac{17pr - 2r^2}{3pr}$, find the value of y when $p = r$.

9. Given that $a = 3$, $b = 4$ and $c = 5$, find the value of

$A = \sqrt{s(s - a)(s - b)(s - c)}$, where $s = \frac{1}{2}(a + b + c)$.

OPERATIONS

Summary:

An operation is a rule connecting two terms using sign language.

EXAMPLES:

1. Given that $a * b = ab^2$, find the value of:

(i) $1 * 3$

(ii) $(1 * 3) * 2$

(iii) a if $a * 3 = 63$

2. Given that $m \uparrow n = \frac{m^2 - n^2}{m + n}$, find the value of:

(i) $5 \uparrow 3$

(ii) $8 \uparrow (5 \uparrow 3)$

(iii) $9 \uparrow (6 \uparrow 2)$

(iv) $(12 \uparrow 4) \uparrow (7 \uparrow 2)$

3. Given that $p * q = \frac{pq}{p + q}$, find the value of:

(i) $3 * (10 * 15)$

(ii) $6 * (20 * 30)$

(iii) $15 * (30 * 15)$

(iv) p if $p * 3 = 2$

4. Given that $a \wedge b = ab - 5$, find the value of y if $3 \wedge y = 6 \wedge 4$

EER:

1. Given that $x \wedge y = x^y - y$, find the value of:

(i) $3 \wedge 2$

(ii) $(3 \wedge 2) \wedge 2$

2. Given that $a * b = \frac{a^2 + b^2 - 2ab}{a - b}$, find the value of:

(i) $4 * 3$

(ii) $8 * (4 * 3)$

3. Given that $a * b = \frac{a + b}{a - b}$, find the value of $(5 * 3) * -2$

4. Given that $m * n = \frac{m^2 + n^2}{10n}$, find the value of:

(i) $4 * -8$

(ii) $7 * (4 * -8)$

5. Given that $a * b = a^2 + \sqrt{ab}$, find the value of $(1 * 4) * 3$

6. Given that $a * b = \frac{a}{b} + \frac{b}{a}$, find the value of $\frac{1}{2} * \frac{2}{3}$

7. Given that $a * b = a^2 + \sqrt{3ab}$, find the value of:

(i) $3 * 4$

(ii) $5 * (3 * 4)$

8. Given that $a * b = a^2 - 2b$, find the value of:

(i) $-3 * -4$

(ii) y if $y * 2 = 21$

9. Given that $a * b = a(b + 2)$, find the value of:

(i) $1 * (2 * 5)$

(ii) y if $4 * y = 16$

COLLECTING LIKE TERMS

Summary

1. In collecting like terms, the expression is re-arranged so that like terms are next to each other.
2. The products xy and yx are the same. Thus xy and yx are like terms.
3. Like terms with powers can be added or subtracted if the powers are the same. Thus $4p + p^2$ cannot be simplified because $4p$ and p^2 are not like terms.

EXAMPLES:

Simplify the following expressions:

(i) $6x + 8 + x^2 - 2x + 2x^2 - 3$

(ii) $5 + 4x^2 - 3 + 2x + 3x^2 - 4x$

(iii) $3x^2 - 7 + 4x^3 - x^2 + 2$

(iv) $4p + 6pq - 2q + 8p - 11qp + 10q$

REMOVING BRACKETS

Summary

1. When removing brackets, each term inside the brackets is multiplied by the quantity outside the brackets.
2. If the sign in front of the bracket is negative, the signs inside the bracket

are changed.

EXAMPLES:

1. Remove the brackets and simplify the following expressions:

(i) $3(x + 2) + 2(x - 4)$ (ii) $2(4x - 2) - (5x - 3)$ (iii) $4(1 - 2x) - 3(6x - 5)$

(iv) $5(3x + 2) - 2(4p + 3)$ (v) $4x(x + 2) + x(x - 4)$ (vi) $3x(5 + 2x) - 2x(3x - 7)$

(vii) $a(b + c) - 2b(c + a) + c(a + b)$ (viii) $4x\left(1 - \frac{2}{x}\right) + 5(2x - 3)$

EER:

1. Remove the brackets and simplify the following expressions:

(i) $2(5a + 3b) + 3(a - 2b)$	(ii) $4(1 - 2x) - 3(3x - 4)$
(iii) $4(1 - 2x) - 3(3x - 4)$	(iv) $3(x + 1) + 2(x + 4)$
(v) $5((4y - 9) - (10y - 5))$	(vi) $9(2x + 3) - 3(5x + 1)$
(vii) $-4(a - 3b) - 3(-3a - b)$	(viii) $2p(q + r) - p(3q - 2r)$
(ix) $2x(x + 3) + x(x - 2)$	(x) $5[4(y - 4) + 15] - [2(5y - 3) + 1]$

SOLVING LINEAR EQUATION

Summary:

1. An equation consists of two expressions separated by an equal sign

2. In solving linear equations the following apply:

(i) When moving any term from one side of the equation to the other, its sign changes

(ii) If the equation contains brackets, first remove the brackets and then workout

(iii) If the equation contains fractions, multiply each term by the **LCM** of the denominators to remove the fractions

EXAMPLES:

1. Solve for x in the following equations:

(i) $6x - 22 = 3x - 10$

(ii) $2(3x - 6) = 3(5 + x)$

(iii) $5 - 2(x - 1) = 4(3 - x) - 2x$

(iv) $(5x - 4) - (3x - 1) = 3$

(v) $(8x + 3) - 3(x - 1) = x - 2$

2. Solve for x in the following equations:

(i) $\frac{3x - 6}{31x - 22} = \frac{1}{13}$

(ii) $\frac{2}{x + 3} = \frac{3}{5 - x}$

3. Given that $(7x - 5) : (2 + 4x) = 8:7$, find the value of x

4. Solve for x in the following equations:

(i) $\frac{2x - 3}{2} - \frac{5 - x}{4} = \frac{x}{3}$

(ii) $\frac{x - 3}{4} + \frac{x - 1}{5} - \frac{x - 2}{3} = 1$

(iii) $\frac{2x - 1}{9} - \frac{x - 1}{4} = 0$

(iv) $\frac{5x - 4}{8} - \frac{x - 3}{5} = \frac{x + 6}{4}$

(v) $\frac{5x - 4}{7} - \frac{1}{2} = \frac{3x - 1}{14}$

(vi) $\frac{3}{5}(5x - 4) = 1 - \frac{2}{5}(x - 1)$

(v) $(x - 1) = \frac{3}{4}(x + 1) - \frac{1}{2}$

(vi) $\frac{1}{3}(x - 1) - \frac{1}{4}(x - 2) = 1$

EER:

1. Solve for x in the following equations:

(i) $4(2x + 3) = 31 - 3(x - 1)$

(ii) $15(x - 7) - 3(x - 9) + 5(x + 6) = 0$

(iii) $10 - 2(x - 4) = 2(x - 1) - 6x$

(iv) $5(x - 1) = 3(2x - 5) - (1 - 3x)$

(v) $9 - 2(x - 5) = x + 10$

2. Solve for x in the following equations:

$$(i) \frac{8x - 5}{7x + 1} = -\frac{4}{5}$$

$$(ii) \frac{7}{2 + 4x} = \frac{8}{7x - 5}$$

3. Given that $(x + 3) : x = 8:5$, find the value of x

4. Given that $(x + 5) : 2 = 18:3$, find the value of x

5. Solve for x in the following equations:

$$(i) \frac{x + 7}{6} - \frac{3x - 2}{3} = \frac{2x + 3}{3}$$

$$(ii) \frac{x - 1}{3} - \frac{x - 2}{4} = 1$$

$$(iii) \frac{3x - 1}{5} - \frac{x}{7} = 3$$

$$(iv) 4 - \frac{x - 3}{2} = 3$$

$$(v) \frac{x}{3} - \frac{x - 2}{2} = \frac{7}{3}$$

$$(vi) \frac{x}{2} = 3 + \frac{x}{3}$$

$$(vii) \frac{x}{2} - \frac{1}{4} = \frac{x}{3} + \frac{1}{2}$$

$$(viii) \frac{x}{2} + \frac{x}{3} = x - 7$$

WORD PROBLEMS ON LINEAR EQUATION

Summary:

In solving word problems on linear equation, read the problem carefully and form an equation using the conditions given in the problem

EXAMPLES:

1. Find two numbers such that one exceeds the other by 9 and their sum is 25

Soln:

If the numbers are x and $(x + 9)$,

$$\Rightarrow x + (x + 9) = 25$$

$$x = 8$$

$$\Rightarrow (x + 9) = 8 + 9 = 17$$

$\therefore$ The numbers are 8 and 17

2. The length of a rectangle is twice its width. If the perimeter of the rectangle is 72m, find the length and width of the rectangle

Soln:

If the width = x

$$\Rightarrow \text{length} = 2x$$

$$\text{If } 2(l + w) = 72$$

$$\Rightarrow 2(2x + x) = 72$$

$$x = 12\text{m}$$

$$\therefore \text{Width} = 12\text{m}, \quad \text{length} = 2(12) = 24\text{m}$$

3. The sum of three consecutive multiples of 5 is 90. Find these multiples

Soln:

If the multiples are x , $(x + 5)$ and $(x + 10)$,

$$\Rightarrow x + (x + 5) + (x + 10) = 90$$

$$x = 25$$

$$\Rightarrow (x + 5) = 25 + 5 = 30, \quad (x + 10) = 25 + 10 = 35$$

$\therefore$ The multiples are 25, 30 and 35

4. Three-fifth of a number is 4 more than one-half of the number. Find the number

Soln:

If the required number = N

$$\Rightarrow \frac{3}{5}N - \frac{1}{2}N = 4$$

$$\therefore N = 40$$

5. *The denominator of a fraction is greater than its numerator by 3. If the numerator is increased by 7 and the denominator is decreased by 1, the fraction becomes $\frac{3}{2}$. Find the original fraction*

Soln:

If the original fraction = $\frac{x}{x+3}$

$$\Rightarrow \frac{x+7}{x+2} = \frac{3}{2}$$

$$\Rightarrow x = 8$$

$$\therefore \text{Original fraction} = \frac{8}{8+3} = \frac{8}{11}$$

6. *In a multiple choice test of 90 questions, each correct answer carries 5 marks and each wrong answer leads to a loss of 2 marks. If a student scored a total of 387 marks from the test, find how many questions were answered correctly*

Soln:

If the correct answers = x , then wrong answers = $(90 - x)$

$$\Rightarrow \text{correct score} - \text{incorrect score} = \text{total score}$$

$$\Rightarrow 5x - 2(90 - x) = 387$$

$$x = 81$$

7. *The length of a rectangle is 4m less than 3 times its width. If the*

perimeter of the rectangle is **32m**, find the length and width of the rectangle

Soln:

If the width = x

$\Rightarrow$ length = $3x - 4$

If $2(l + w) = 32$

$\Rightarrow 2(3x - 4 + x) = 32$

$x = 5m$

$\therefore$ Width = **5**, length = $3(5) - 4 = \mathbf{11m}$

8. A man is **24** years older than his son. In two years time, his age will be twice the age of his son. Find the present age of the son

Soln:

Son's present age = x , Father's present age = $x + 24$

$\Rightarrow$ After two years, Son's age = $x + 2$, Father's age = $x + 26$

$\Rightarrow x + 26 = 2(x + 2)$

$\therefore x = 22$ years

9. Tom's present age is two-fifth of his father's age. After **8** years he will be one-half of his father's age then. Find how old is his father

Soln:

Father's present age = x , Tom's present age = $\frac{2}{5}x$

$\Rightarrow$ After **8** years, Father's age = $x + 8$ Tom's age = $\frac{2}{5}x + 8$

$\Rightarrow \frac{2}{5}x + 8 = \frac{1}{2}(x + 8)$

$\therefore x = 40$ years

10. The sum of the present ages of a man and his son is **60** years. Six years ago, the man's age was **5** times the son's age. Find the son's present age

Soln:

Son's present age = x , Father's present age = $60 - x$

$\Rightarrow$ Six years ago, Son's age = $x - 6$, Father's age = $(60 - x) - 6 = 54 - x$

$\Rightarrow 54 - x = 5(x - 6)$

$\therefore x = 14$ years

11. The sum of the present ages of a man and his son is **45** years. Five years ago, the product of their ages was **4** times the man's age at that time. Find their present ages

Soln:

Son's present age = x , Father's present age = $45 - x$

$\Rightarrow$ Five years ago, Son's age = $x - 5$, Father's age = $(45 - x) - 5 = 40 - x$

$\Rightarrow (x - 5)(40 - x) = 4(40 - x)$

$\Rightarrow (x - 5) = 4$

$\Rightarrow x = 9$ years

$\therefore$ Son's age = **9 years**, Father's age = $45 - 9 = 36$ years

12. Ten years ago, a man's age was thrice as old as his son. In ten years time, the man's age will be twice as old as his son. Find their present ages

Soln:

Ten years ago, Son's age = x , Father's age = $3x$

$\Rightarrow$ Son's present age = $x + 10$, Father's present age = $3x + 10$

After 10 years, Son's age = $x + 20$, Father's age = $3x + 20$

$\Rightarrow 3x + 20 = 2(x + 20)$

$$\Rightarrow x = 20 \text{ years}$$

$\therefore$ Son's present age = $20 + 10 = 30$ years, Father's age = $3(20) + 10 = 70$ years

13. Tom is 9 years older than Bob. In ten years time Tom will be twice as old as Bob was 10 years ago. Find Bob's present age

Soln:

Bob's present age = x , Tom's present age = $x + 9$

$\Rightarrow$ In ten years, Tom's age = $(x + 9) + 10 = x + 19$,

Ten years ago, Bob's age = $(x - 10)$

$$\Rightarrow x + 19 = 2(x - 10)$$

$$\therefore x = 39 \text{ years}$$

14. A man said to his son, "I was as old as you are at present at the time of your birth". If the man's age is 38 years now, find the son's age 5 years ago

Soln:

Son's present age = x ,

x years ago, father's age = $38 - x$

$$\Rightarrow 38 - x = x$$

$$\Rightarrow x = 19 \text{ years}$$

$$\therefore 5 \text{ years ago, son's age} = 19 - 5 = 14 \text{ years}$$

15. The ratio between two numbers is 7:5. If the difference between them is 48, find the numbers

Soln:

If the common ratio is x ,

$\Rightarrow$ The ratio $7 : 5 = 7x : 5x$ (The ratio $7x : 5x$ simplifies to $7 : 5$)

$\Rightarrow$ The numbers are $7x$ and $3x$

$$\Rightarrow 7x - 3x = 48$$

$$x = 12$$

$$\therefore 7x = 7(12) = 84, \quad 3x = 3(12) = 36$$

Thus the numbers are **84** and **36**

16. Tom is 6 years older than Bob. The ratio between the present ages of Tom and Bob is 7:5. Find their present ages

Soln:

The ratio 7 : 5 = $7x:5x$

$\Rightarrow$ Tom's present age = $7x$, Bob's present age = $5x$

$$\Rightarrow 7x - 5x = 6$$

$$x = 3$$

$$\therefore \text{Tom's age} = 7(3) = 21 \text{ years}, \quad \text{Bob's age} = 5(3) = 15 \text{ years}$$

METHOD II

Bob's age = x

Tom's age = $x + 6$

$$\Rightarrow \frac{x + 6}{x} = \frac{7}{5}$$

$$\Rightarrow x = 15$$

$$\therefore \text{Bob's age} = 15 \text{ years}, \quad \text{Tom's age} = 15 + 6 = 21 \text{ years}$$

17. The present ages of Tom and Bob are in the ratio of 5:4. In three years time, the ratio of their ages will become 11:9 respectively. Find Bob's present age

Soln:

The ratio $5 : 4 = 5x : 4x$

$\Rightarrow$ Tom's present age = $5x$, Bob's present age = $4x$

$\Rightarrow$ After 3 years, Tom's age = $5x + 3$, Bob's age = $4x + 3$

$$\Rightarrow \frac{5x + 3}{4x + 3} = \frac{11}{9}$$

$$\Rightarrow x = 6$$

$\therefore$ Bob's present age = $4(6) = 24$ years

18. Four years ago, the ratio of the ages of Tom and Bob was $3:2$. In six years time, the ratio of their ages will become $14:11$ respectively. Find Bob's present age

Soln:

Four years ago, Tom's age = $3x$, Bob's age = $2x$

$\Rightarrow$ Tom's present age = $3x + 4$, Bob's present age = $2x + 4$

After 6 years, Tom's age = $(3x + 4) + 6 = 3x + 10$,

Bob's age = $(2x + 4) + 6 = 2x + 10$

$$\Rightarrow \frac{3x + 10}{2x + 10} = \frac{14}{11}$$

$$\Rightarrow x = 6$$

$\therefore$ Bob's present age = $2(6) + 4 = 16$ years

19. The present ages of Tom, Bob and Ben are in the ratio $4:7:9$. Eight years ago, the sum of their ages was 76. Find their present ages

Soln:

Tom's present age = $4x$, Bob's age = $7x$, Ben's age = $9x$

$\Rightarrow$ 8 years ago, Tom's age = $4x - 8$, Bob's age = $7x - 8$, Ben's age = $9x - 8$

$$\Rightarrow (4x - 8) + (7x - 8) + (9x - 8) = 76$$

$$\Rightarrow x = 5$$

$\therefore$ Tom's present age = $4(5) = 20$ years, Bob's age = $7(5) = 35$ years

Ben's age = $9(5) = 45$ years

20. Ten years ago, the ratio of the ages of Tom, Bob and Ben was **2:3:7**. The sum of their present ages is **90**. Find Bob's present age

Soln:

Ten years ago, Tom's age = $2x$, Bob's age = $3x$, Ben's age = $7x$

$\Rightarrow$ Tom's present age = $2x + 10$, Bob's age = $3x + 10$, Ben's age = $7x + 10$

$$\Rightarrow (2x + 10) + (3x + 10) + (7x + 10) = 90$$

$$\Rightarrow x = 5$$

$\therefore$ Bob's present age = $3(5) + 10 = 25$ years

EER:

1. Find two numbers such that one exceeds the other by **11** and their sum is **73**

2. The sum of three consecutive multiples of **4** is **444**. Find these multiples

3. The cost of two tables and three chairs is **Shs 705,000**. If the table costs **Shs 40,000** more than the chair, find the cost of each table and a chair

4. The length of a rectangle is **4** times its width. If the perimeter of the rectangle is **80m**, find the length and width of the rectangle

5. Three-fourth of a number is more than one-fourth of a number by 2. Find the number
6. The numerator of a fraction is 5 less than its denominator. If 2 is added to both the numerator and denominator, the fraction becomes $\frac{4}{5}$. Find the original fraction
7. In a multiple choice test of 30 questions, each correct answer carries 5 marks and each wrong answer leads to a loss of 2 marks. If a student scored a total of 80 marks from the test, find how many questions were answered correctly
8. The numerator of a fraction is 1 less than its denominator. If the numerator is decreased by 2 and the denominator is increased by 3, the fraction becomes $\frac{1}{4}$. Find the original fraction
9. The numerator of a fraction is 3 less than its denominator. If the numerator is tripled and the denominator is increased by 7, the resulting fraction is $\frac{3}{2}$. Find the original fraction
10. The sum of the ages of 5 children born at intervals of 3 years each is 50 years. Find the age of the youngest child
11. The present ages of P, Q and R are in the ratio 4:7:9. Eight years ago, the sum of their ages was 56. Find their present ages
12. A man is 4 times as old as his son. Five years ago, the man was 9 times as old as his son. Find the man's present age
13. Tom is 4 times as old as Bob. In 4 years time, the sum of their ages will be 43 years. Find Tom's present age
14. Five years ago, a man's age was seven times as old as his son. In five years time, the man's age will be thrice as old as his son. Find their present ages

- 15.** In 15 years time, Tom's age will be 6 times his age 5 years back. Find his present age
- 16.** Tom is 12 years younger than Bob. The ratio between the present ages of Tom and Bob is 2:5. Find their present ages
- 17.** Ten years ago, P was twice as old as Q. The ratio between the present ages of P and Q is 4:3. Find their present age
- 18.** Six years ago, Tom was thrice as old as Bob. In six years time, Tom will be one and a third times as old as Bob. Find Bob's present age
- 19.** The present ages of Tom and Bob are in the ratio 4:3. In six years time, Tom's age will be 26 years. Find Bob's present age
- 20.** The present ages of Tom and Bob are in the ratio 11:5. If the difference between their ages is 24 years, find their present ages
- 21.** The present ages of a man and his son are in the ratio 4:1. If the product of their ages is 196 years, find the ratio of their ages in 5 years time
- 22.** The ratio between the present ages of P and Q is 2:5. In eight years time, their ages will be in the ratio 1:2. Find their present ages
- 23.** Six years ago, the ratio of the ages of Tom and Bob was 6:5. In four years time, the ratio of their ages will become 11:10 respectively. Find Bob's present age
- 24.** The ages of two people differ by 16 years. Six years ago, the elder one was 3 times as old as the younger one. Find the elder's present age

ALGEBRAIC FRACTIONS

Summary:

(i) A fraction with any letter in its numerator or denominator is called an algebraic fraction

(ii) A fraction is undefined (**meaningless**) when its denominator is equal to zero.

(iii) To reduce an algebraic fraction in its simplest form, factorization is sometimes necessary

EXAMPLES:

1. Find the value of x that makes the fraction $\frac{5x + 9}{4x - 12}$ undefined

2. Find the values of x for which the fraction $\frac{3x + 4}{x^2 - 25}$ is undefined

3. Find the values of x for which the fraction $\frac{5x}{2x^2 - 18}$ is undefined

4. Express the following as a single fraction:

(i) $\frac{x}{3} + \frac{x}{4} + \frac{x}{5}$

(ii) $\frac{x - 2}{4} + \frac{2}{5}$

(iii) $\frac{3x - 5}{10} + \frac{2x - 3}{15}$

(iv) $\frac{x - 5}{3} - \frac{x - 2}{4}$

(v) $3(x + 2) - \frac{4x - 5}{4}$

(vi) $\frac{2x - 5}{5} - \frac{3x + 2}{4} + \frac{7x + 15}{10}$

5. Reduce the following fractions to their lowest form:

(i) $\frac{3xy^2}{6x^2y}$

(ii) $\frac{18x^2y}{27xy^2z}$

(iii) $\frac{(x + 5)(x - 1)}{(x + 3)(x + 5)}$

(iv)

$\frac{(2x - 1)^5}{(2x - 1)^3}$

(v) $\frac{x^5(x + 1)^2}{x^3(x + 1)^8}$

(vi) $\frac{12x}{20x + 8}$

(vii) $\frac{27x + 15}{9x}$

(viii) $\frac{3x - 6}{(x + 5)(x - 2)}$

$$(ix) \frac{2x + 4}{3x + 6}$$

$$(x) \frac{5 - 3x}{3x - 5}$$

$$(xi) \frac{3x}{x^2 - 5x}$$

$$(xii) \frac{5x - x^2}{(x + 2)(x - 5)}$$

$$(xiii) \frac{(x + 4)(x - 3)}{3x - x^2}$$

$$(xiv) \frac{12abc - 4a^2bc^2}{8abc^2}$$

(i) Soln:

$$\frac{3xy^2}{6x^2y} = \frac{3x \cdot y \cdot y}{6x \cdot x \cdot y} = \frac{y}{2x}$$

6. Simplify the following fractions as far as possible:

$$(i) \frac{3a^2b}{2b^3c} \times \frac{2c^2}{5a^3} \div \frac{6ac}{10b^2}$$

$$(ii) \frac{a^2}{(2a)^2} \times \frac{(3b)^2}{27a} \div 3b$$

$$(iii) \frac{7a}{2a + 14} \times \frac{3a + 21}{9a}$$

$$(iv) \frac{y^2 + 3y}{6y^2} \div \frac{5y + 15}{4}$$

LCM OF ALGEBRAIC EXPRESSIONS

Summary:

The **LCM** of given expressions is obtained as follows:

(i) Determine the prime factor form of each expression

(ii) The product of each factor with the highest power is the required **LCM**

EXAMPLES:

1. Find the **LCM** of $16y^3$ and $24y^5$

Soln:

$$16y^3 = 2^4 \times y^3$$

$$24y^5 = 2^3 \times 3 \times y^5$$

$$\begin{aligned} LCM &= 2^4 \times 3 \times y^5 \\ &= 48y^5 \end{aligned}$$

2. Find the LCM of $8a^2bc^3$ and $20ac^4$

Soln:

$$8a^2bc^3 = 2^3 \times a^2 \times b \times c^3$$

$$20ac^4 = 2^2 \times 5 \times a \times c^4$$

$$\begin{aligned} LCM &= 2^3 \times 5 \times a^2 \times b \times c^4 \\ &= 40a^2bc^4 \end{aligned}$$

3. Find the LCM of $4a^2b$, $6ab$ and $8ab^3$

Soln:

$$4a^2b = 2^2 \times a^2 \times b$$

$$6ab = 2 \times 3 \times a \times b$$

$$8ab^3 = 2^3 \times a \times b^3$$

$$\begin{aligned} LCM &= 2^3 \times 3 \times a^2 \times b^3 \\ &= 24a^2b^3 \end{aligned}$$

4. Find the LCM of $y(y + 2)$ and $(y + 2)(y - 3)$

Soln:

$$y(y + 2) = y \times (y + 2)$$

$$(y + 2)(y - 3) = (y + 2) \times (y - 3)$$

$$\begin{aligned} LCM &= y \times (y + 2) \times (y - 3) \\ &= y(y + 2)(y - 3) \end{aligned}$$

5. Find the LCM of $(y - 5)(y + 3)$ and $(y + 3)^2$

Soln:

$$(y - 5)(y + 3) = (y - 5) \times (y + 3)$$

$$(y + 3)^2 = 1 \times (y + 3)^2$$

$$\begin{aligned} LCM &= (y - 5) \times (y + 3)^2 \\ &= (y - 5)(y + 3)^2 \end{aligned}$$

6. Find the LCM of $ab(a - b)$, $4(a - b)$ and $6(a - b)^2$

Soln:

$$ab(a - b) = a \times b \times (a - b)$$

$$4(a - b) = 2^2 \times (a - b)$$

$$6(a - b)^2 = 2 \times 3 \times (a - b)^2$$

$$\begin{aligned} LCM &= 2^2 \times 3 \times a \times b \times (a - b)^2 \\ &= 12ab(a - b)^2 \end{aligned}$$

7. Simplify the following fractions as far as possible:

$$\begin{array}{lll}
 \text{(i)} \frac{2}{3x} + \frac{1}{2x} + \frac{3}{4x} & \text{(ii)} \frac{4}{5x} + \frac{2}{x^2} - \frac{1}{3x} & \text{(iii)} 2 + \frac{3}{x+1} \\
 \text{(iv)} x - \frac{5}{x} & \text{(v)} \frac{5}{x-3} - \frac{1}{x} & \text{(vi)} \\
 \frac{3}{x+2} + \frac{2}{x-1} & \text{(vii)} \frac{5x}{x-1} - \frac{2x}{x+1} & \text{(viii)} \frac{13x+1}{2x+1} - \frac{7x-2}{2x+1} \quad \text{(ix)} \\
 \frac{2x^2}{x+4} + \frac{8}{x+4} & \text{(x)} \frac{9-5x}{3x-2} - \frac{7-2x}{3x-2} & \text{(xi)} \\
 \frac{3x}{(x-2)(x+5)} - \frac{6}{(x-2)(x+5)} & \text{(xii)} \frac{2}{x(x+2)} + \frac{3}{(x+2)(x-3)} \\
 \text{(xiii)} \frac{2x}{(x-3)(x+3)} - \frac{1}{(x+3)} - \frac{2}{(x-3)}
 \end{array}$$

EER:

- Find the values of x that make the fraction $\frac{14x}{x^2 - 9}$ undefined
- Find the values of x for which the fraction $\frac{5x+3}{9x^2 - 4}$ is undefined
- Find the values of x for which the fraction $\frac{10x}{x^2 - 4}$ is undefined
- Find the **LCM** of $6a^2b^3c^2$, $20ab^2c^5$ and $18a^4b^2c$
- Find the **LCM** of $ab(a-b)^2$, $6a^2(a-b)$ and $8(a-b)^3$
- Simplify as far as possible: $\frac{6x^2y}{13p^2q} \times \frac{39q^2r}{18y^2z} \div \frac{x^2p^2}{yqr}$
- Express $1 - \frac{2x}{5} + \frac{x}{8}$ as a single fraction
- Express $\frac{x+3}{3} - \frac{x-7}{4} + \frac{x-5}{2}$ as a single fraction

9. Express $4x - \left(\frac{3x + y}{8} - \frac{3x + y}{4} \right)$ as a single fraction

10. Express $\frac{3}{x+1} - \frac{x+4}{(x+1)(x-2)}$ as a single fraction

11. Given that $y = \frac{7}{x-2} - \frac{x+5}{(x-1)(x-2)}$, express y in the form $\frac{a}{x+b}$.

Hence find the value of x for which y is undefined

12. Express $\frac{2}{x+5} + \frac{1}{x-3} - \frac{5}{(x+5)(x-3)}$ as a single fraction

13. Express $\frac{4}{x+5} + \frac{1}{x-2} - \frac{7}{(x+5)(x-2)}$ as a single fraction

14. Reduce the following fractions to their lowest form:

(i) $\frac{14a^2bc}{42ab^3c^4}$ (ii) $\frac{-6mn^4}{-24m^5n^2}$ (iii) $\frac{(x+3)(x-7)}{6(x+3)}$ (iv) $\frac{4(p+7)}{(p+7)^3}$

(v) $\frac{x+2}{(x+5)(x+2)}$ (vi) $\frac{6m}{m^2-4m}$ (vii) $\frac{2x+10}{x^2+5x}$ (viii) $\frac{x-5}{5-x}$

(ix) $\frac{3x-x^2}{(x+5)(x-3)}$ (xi) $\frac{15a^2bc^2-12abc}{18abc^2}$

15. Simplify the following fractions as far as possible:

(i) $\frac{x-1}{x+2} \div \frac{5(x-1)}{x+3}$

(ii) $\frac{14a^2bc}{42ab^3c^4}$

(iii) $\frac{3a}{6a+6b} \times \frac{a+b}{5a^2}$

(iv) $\frac{5y+25}{y^3} \div \frac{2y+10}{7y}$

16. Simplify the following fractions as far as possible:

(i) $\frac{1}{x} + \frac{2}{x^2} - \frac{1}{x^3}$

(ii) $3 + \frac{2}{x+2}$

(iii) $x - \frac{5}{x}$

(iv) $\frac{3}{x-3} - \frac{2}{x}$

(v) $\frac{6}{x-1} + \frac{3}{x+1}$

(vi) $\frac{5x}{x-1} - \frac{2x}{x+1}$

$$(vii) \frac{6x+1}{x-3} - \frac{4x-2}{x-3} \quad (viii) \frac{3x^2}{x+2} + \frac{6}{x+2} \quad (ix) \frac{9-5x}{3x-2} - \frac{7-2x}{3x-2}$$